Finite Element Analysis (4th Edition)

Chapter 3, Problem 5P

Step-by-step solution

Step 1 of 27

Consider each truss member as element and each joint connecting them as nodes. Hence, the specified truss has 2 element and 4 nodes and are tabulated as follows.

Element	Node i	Node j	θ
(1)	1	2	$0°$
(2)	2	3	$(90+56.31)° = 146.31°$
(3)	3	4	$0°$
(4)	2	4	$90°$

Here, the angle measured in counter clockwise direction.

Comment

Step 2 of 27

Convert the units of area from cm^2 to mm^2.

$$A = 15\ cm^2 = 1500\ mm^2$$

Calculate the equivalent stiffness constant for element (1) of the truss by using the following relation.

$$k_1 = \frac{AE}{L}$$

Here, the area of cross section is A, E is the modulus of elasticity, and L is the length of the member.

Substitute $1500\ mm^2$ for A, $70000\ N/mm^2$ for E, and $1500\ mm$ for L.

$$k_1 = \frac{AE}{L} = \frac{1500 \times 70 \times 10^3}{1500} = 70 \times 10^3\ N/mm$$

Calculate the equivalent stiffness constant for element (3) of the truss by using the following relation.

$$k_3 = \frac{AE}{L}$$

Here, the area of cross section is A, E is the modulus of elasticity, and L is the length of the member.

Substitute $1500\ mm^2$ for A, $70000\ N/mm^2$ for E, and $1500\ mm$ for L.

$$k_3 = \frac{AE}{L} = \frac{1500 \times 70 \times 10^3}{1500} = 70 \times 10^3\ N/mm$$

Comment

Step 3 of 27

Find the length of the element 2.

$$L = \sqrt{1.5^2 + 1^2} = 1.80278\ m = 1802.78\ mm = 1803\ mm$$

Calculate the equivalent stiffness constant for element (2) of the truss by using the following relation.

$$k_2 = \frac{AE}{L}$$

Here, the area of cross section is A, E is the modulus of elasticity, and L is the length of the member.

Substitute $1500\ mm^2$ for A, $70000\ N/mm^2$ for E, and $1803\ mm$ for L.

$$k_2 = \frac{AE}{L} = \frac{1500 \times 70 \times 10^3}{1803} = 58.243 \times 10^3\ N/mm$$

Calculate the equivalent stiffness constant for element (4) of the truss by using the following relation.

$$k_4 = \frac{AE}{L}$$

Here, the area of cross section is A, E is the modulus of elasticity, and L is the length of the member.

Substitute $1500\ mm^2$ for A, $70000\ N/mm^2$ for E, and $1000\ mm$ for L.

$$k_4 = \frac{AE}{L} = \frac{1500 \times 70 \times 10^3}{1000} = 105 \times 10^3\ N/mm$$

Comment

Step 4 of 27

Find the local coordinate stiffness matrix element (1).

$$[\mathbf{K}]^1 = k_1 \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta & -\cos^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta & -\sin\theta\cos\theta & -\sin^2\theta \\ -\cos^2\theta & -\sin\theta\cos\theta & \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & -\sin^2\theta & \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

Substitute $70 \times 10^3\ N/mm$ for k_1 and $0°$ for θ in the equation.

$$[\mathbf{K}]^1 = k_1 \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta & -\cos^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta & -\sin\theta\cos\theta & -\sin^2\theta \\ -\cos^2\theta & -\sin\theta\cos\theta & \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & -\sin^2\theta & \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$= 70 \times 10^3 \begin{bmatrix} \cos^2(0°) & \sin(0°)\cos(0°) & -\cos^2(0°) & -\sin(0°)\cos(0°) \\ \sin(0°)\cos(0°) & \sin^2(0°) & -\sin(0°)\cos(0°) & -\sin^2(0°) \\ -\cos^2(0°) & -\sin(0°)\cos(0°) & \cos^2(0°) & \sin(0°)\cos(0°) \\ -\sin(0°)\cos(0°) & -\sin^2(0°) & \sin(0°)\cos(0°) & \sin^2(0°) \end{bmatrix}$$

$$= 70 \times 10^3 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \end{matrix}$$

$$= 10^3 \begin{bmatrix} 70 & 0 & -70 & 0 \\ 0 & 0 & 0 & 0 \\ -70 & 0 & 70 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \end{matrix}$$

Comment

Step 5 of 27

Find the global coordinate system for Stiffness matrix equation for the element (1).

$$[\mathbf{K}]^{1G} = 10^3 \begin{bmatrix} 70 & 0 & -70 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -70 & 0 & 70 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \\ U_{4Y} \\ U_{4Y} \end{matrix}$$

Comment

Step 6 of 27

Find the local coordinate stiffness matrix element (2).

$$[\mathbf{K}]^2 = k_2 \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta & -\cos^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta & -\sin\theta\cos\theta & -\sin^2\theta \\ -\cos^2\theta & -\sin\theta\cos\theta & \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & -\sin^2\theta & \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

Substitute $58.243 \times 10^3\ N/mm$ for k_2 and $146.31°$ for θ in the equation.

$$[\mathbf{K}]^2 = k_2 \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta & -\cos^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta & -\sin\theta\cos\theta & -\sin^2\theta \\ -\cos^2\theta & -\sin\theta\cos\theta & \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & -\sin^2\theta & \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$= 58.24 \times 10^3 \begin{bmatrix} \cos^2(146.31°) & \sin(146.31°)\cos(146.31°) & -\cos^2(146.31°) & -\sin(146.31°)\cos(146.31°) \\ \sin(146.31°)\cos(146.31°) & \sin^2(146.31°) & -\sin(146.31°)\cos(146.31°) & -\sin^2(146.31°) \\ -\cos^2(146.31°) & -\sin(146.31°)\cos(146.31°) & \cos^2(146.31°) & \sin(146.31°)\cos(146.31°) \\ -\sin(146.31°)\cos(146.31°) & -\sin^2(146.31°) & \sin(146.31°)\cos(146.31°) & \sin^2(146.31°) \end{bmatrix}$$

$$= 58.24 \times 10^3 \begin{bmatrix} 0.6923 & -0.4615 & -0.6923 & 0.4615 \\ -0.4615 & 0.3076 & 0.4615 & -0.3076 \\ -0.6923 & 0.4615 & 0.6923 & -0.4615 \\ 0.4615 & -0.3076 & -0.4615 & 0.3076 \end{bmatrix} \begin{matrix} U_{2x} \\ U_{2y} \\ U_{3x} \\ U_{3y} \end{matrix}$$

$$= 10^3 \begin{bmatrix} 40.32 & -26.88 & -40.32 & 26.88 \\ -26.88 & 17.92 & 26.88 & -17.92 \\ -40.32 & 26.88 & 40.32 & -26.88 \\ 26.88 & -17.92 & -26.88 & 17.92 \end{bmatrix} \begin{matrix} U_{2x} \\ U_{2y} \\ U_{3x} \\ U_{3y} \end{matrix}$$

Comment

Step 7 of 27

Find the global coordinate system for Stiffness matrix equation for the element (2).

$$[\mathbf{K}]^{2G} = 10^3 \times \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 40.32 & -26.88 & -40.32 & 26.88 & 0 & 0 \\ 0 & 0 & -26.88 & 17.92 & 26.88 & -17.92 & 0 & 0 \\ 0 & 0 & -40.32 & 26.88 & 40.32 & -26.88 & 0 & 0 \\ 0 & 0 & 26.88 & -17.92 & -26.88 & 17.92 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \\ U_{4Y} \\ U_{4Y} \end{matrix}$$

Comment

Step 8 of 27

Find the local coordinate stiffness matrix element (3).

$$[\mathbf{K}]^3 = k_3 \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta & -\cos^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta & -\sin\theta\cos\theta & -\sin^2\theta \\ -\cos^2\theta & -\sin\theta\cos\theta & \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & -\sin^2\theta & \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

Substitute $70 \times 10^3\ N/mm$ for k_3 and $0°$ for θ in the equation.

$$[\mathbf{K}]^3 = k_3 \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta & -\cos^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta & -\sin\theta\cos\theta & -\sin^2\theta \\ -\cos^2\theta & -\sin\theta\cos\theta & \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & -\sin^2\theta & \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$= 70 \times 10^3 \begin{bmatrix} \cos^2(0°) & \sin(0°)\cos(0°) & -\cos^2(0°) & -\sin(0°)\cos(0°) \\ \sin(0°)\cos(0°) & \sin^2(0°) & -\sin(0°)\cos(0°) & -\sin^2(0°) \\ -\cos^2(0°) & -\sin(0°)\cos(0°) & \cos^2(0°) & \sin(0°)\cos(0°) \\ -\sin(0°)\cos(0°) & -\sin^2(0°) & \sin(0°)\cos(0°) & \sin^2(0°) \end{bmatrix}$$

$$= 70 \times 10^3 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} U_{3x} \\ U_{3y} \\ U_{4x} \\ U_{4y} \end{matrix}$$

$$= 10^3 \begin{bmatrix} 70 & 0 & -70 & 0 \\ 0 & 0 & 0 & 0 \\ -70 & 0 & 70 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} U_{3x} \\ U_{3y} \\ U_{4x} \\ U_{4y} \end{matrix}$$

Comment

Step 9 of 27

Find the global coordinate system for Stiffness matrix equation for the element (3).

$$[\mathbf{K}]^{3G} = 10^3 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 70 & 0 & -70 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -70 & 0 & 70 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \\ U_{4Y} \\ U_{4Y} \end{matrix}$$

Comment

Step 10 of 27

Find the local coordinate stiffness matrix element (4).

$$[\mathbf{K}]^4 = k_4 \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta & -\cos^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta & -\sin\theta\cos\theta & -\sin^2\theta \\ -\cos^2\theta & -\sin\theta\cos\theta & \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & -\sin^2\theta & \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

Substitute $58.243 \times 10^3\ N/mm$ for k_4 and $90°$ for θ in the equation.

$$[\mathbf{K}]^4 = k_4 \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta & -\cos^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta & -\sin\theta\cos\theta & -\sin^2\theta \\ -\cos^2\theta & -\sin\theta\cos\theta & \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & -\sin^2\theta & \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$= 105 \times 10^3 \begin{bmatrix} \cos^2(90°) & \sin(90°)\cos(90°) & -\cos^2(90°) & -\sin(90°)\cos(90°) \\ \sin(90°)\cos(90°) & \sin^2(90°) & -\sin(90°)\cos(90°) & -\sin^2(90°) \\ -\cos^2(90°) & -\sin(90°)\cos(90°) & \cos^2(90°) & \sin(90°)\cos(90°) \\ -\sin(90°)\cos(90°) & -\sin^2(90°) & \sin(90°)\cos(90°) & \sin^2(90°) \end{bmatrix}$$

$$= 105 \times 10^3 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{matrix} U_{2x} \\ U_{2y} \\ U_{4x} \\ U_{4y} \end{matrix}$$

Comment

Step 11 of 27

Find the global coordinate system for Stiffness matrix equation for the element (4).

$$[\mathbf{K}]^{4G} = 10^3 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 105 & 0 & 0 & 0 & -105 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -105 & 0 & 0 & 0 & 105 \end{bmatrix} \begin{matrix} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \\ U_{4Y} \\ U_{4Y} \end{matrix}$$

Find the global stiffness matrix of the truss using the relation.

$$[\mathbf{K}]^G = [\mathbf{K}]^{1G} + [\mathbf{K}]^{2G} + [\mathbf{K}]^{3G} + [\mathbf{K}]^{4G}$$

$$= 10^3 \times \begin{bmatrix} 70 & 0 & -70 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -70 & 0 & 110.32 & -26.88 & -40.32 & 26.88 & 0 & 0 \\ 0 & 0 & -26.88 & 122.92 & 26.88 & -17.92 & 0 & -105 \\ 0 & 0 & -40.32 & 26.88 & -29.67 & -26.88 & 0 & 0 \\ 0 & 0 & 26.88 & -17.92 & -26.8 & 17.92 & 0 & 0 \\ 0 & 0 & 0 & 0 & -70 & 0 & 70 & 0 \\ 0 & 0 & 0 & -105 & 0 & 0 & 0 & 105 \end{bmatrix}$$

Comment

Step 12 of 27

Apply the boundary condition and loads.

As the nodes 1 and 3 are hinged, the following conditions are obtained.

$$U_{1X} = 0$$
$$U_{1Y} = 0$$
$$U_{3X} = 0$$
$$U_{3Y} = 0$$

Comment

Step 13 of 27

Write the basic governing equation.

$$[\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{f}\}$$

Here, $[\mathbf{K}]$ is the global stiffness matrix, $\{\mathbf{u}\}$ is the displacement matrix, and $\{\mathbf{f}\}$ is the force matrix.

Substitute $10^3 \times \begin{bmatrix} 70 & 0 & -70 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -70 & 0 & 110.32 & -26.88 & -40.32 & 26.88 & 0 & 0 \\ 0 & 0 & -26.88 & 122.92 & 26.88 & -17.92 & 0 & -105 \\ 0 & 0 & -40.32 & 26.88 & -29.67 & -26.88 & 0 & 0 \\ 0 & 0 & 26.88 & -17.92 & -26.8 & 17.92 & 0 & 0 \\ 0 & 0 & 0 & 0 & -70 & 0 & 70 & 0 \\ 0 & 0 & 0 & -105 & 0 & 0 & 0 & 105 \end{bmatrix}$ for $[\mathbf{K}]$, $\begin{Bmatrix} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \\ U_{4Y} \\ U_{4Y} \end{Bmatrix}$ for $\{\mathbf{u}\}$, and $\begin{Bmatrix} F_{1X} \\ F_{1Y} \\ F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \\ F_{4X} \\ F_{4Y} \end{Bmatrix}$ for $\{\mathbf{f}\}$.

$$10^3 \begin{bmatrix} 70 & 0 & -70 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -70 & 0 & 110.32 & -26.88 & -40.32 & 26.88 & 0 & 0 \\ 0 & 0 & -26.88 & 122.92 & 26.88 & -17.92 & 0 & -105 \\ 0 & 0 & -40.32 & 26.88 & -29.67 & -26.88 & 0 & 0 \\ 0 & 0 & 26.88 & -17.92 & -26.8 & 17.92 & 0 & 0 \\ 0 & 0 & 0 & 0 & -70 & 0 & 70 & 0 \\ 0 & 0 & 0 & -105 & 0 & 0 & 0 & 105 \end{bmatrix} \begin{Bmatrix} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \\ U_{4Y} \\ U_{4Y} \end{Bmatrix} = \begin{Bmatrix} F_{1X} \\ F_{1Y} \\ F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \\ F_{4X} \\ F_{4Y} \end{Bmatrix}$$

Comment

Step 14 of 27

Substitute 0 for U_{1X}, 0 for U_{1Y}, 0 for U_{3X}, and 0 for U_{3Y}, 0 for F_{1X}, 0 for F_{1Y}, 0 for F_{2X}, 0 for F_{3X}, 0 for F_{3Y}, and $-2000\ N$ for F_{4Y} in equation.

$$10^3 \begin{bmatrix} 70 & 0 & -70 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -70 & 0 & 110.32 & -26.88 & -40.32 & 26.88 & 0 & 0 \\ 0 & 0 & -26.88 & 122.92 & 26.88 & -17.92 & 0 & -105 \\ 0 & 0 & -40.32 & 26.88 & -29.67 & -26.88 & 0 & 0 \\ 0 & 0 & 26.88 & -17.92 & -26.8 & 17.92 & 0 & 0 \\ 0 & 0 & 0 & 0 & -70 & 0 & 70 & 0 \\ 0 & 0 & 0 & -105 & 0 & 0 & 0 & 105 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ U_{2X} \\ U_{2Y} \\ 0 \\ 0 \\ U_{4X} \\ U_{4Y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2000 \end{bmatrix}$$

$$10^3 \begin{bmatrix} -70U_{2X} \\ 0 \\ 110.32U_{2X} - 26.88U_{2Y} \\ -26.88U_{2X} + 122.92U_{2Y} - 105U_{4Y} \\ -40.322U_{2X} + 26.88U_{2Y} \\ 26.88U_{2X} - 17.92U_{2Y} \\ 70U_{4X} \\ -105U_{2Y} + 105U_{4Y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2000 \end{bmatrix}$$

$$10^3 \begin{bmatrix} 110.32U_{2X} - 26.88U_{2Y} \\ -26.88U_{2X} + 122.92U_{2Y} - 105U_{4Y} \\ 70U_{4X} \\ -105U_{2Y} + 105U_{4Y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2000 \end{bmatrix} \quad \text{......(1)}$$

Comment

Step 15 of 27

Solve the matrix to obtain the deflections in the truss.

Consider the following relations from matrix (1).

Consider the third row

$$-70U_{4X} = 0$$
$$U_{4X} = 0$$

Therefore, the x component of the deflection at joint 4 is $\boxed{0}$.

Comment

Step 16 of 27

Consider the fourth row

$$-105U_{4Y} + 105U_{2Y} = 2000$$
$$105U_{4Y} = -2000 + 105U_{2Y}$$
$$U_{4Y} = \frac{-2000 + 105U_{2Y}}{105 \times 1000} \quad \text{...... (2)}$$

Consider the first row

$$110.32U_{2X} - 26.88U_{2Y} = 0$$
$$U_{2X} = \frac{26.88U_{2Y}}{110.322} \quad \text{...... (3)}$$

Comment

Step 17 of 27

Consider the second row

$$-26.88U_{2X} + 122920.97U_{2Y} - 105U_{4Y} = 0$$

Substitute the equations (2) and (3) in the equation.

$$-26.88 \times 1000 \times \left(\frac{26.88}{110.322}\right) + 122920.97U_{2Y} - 105000 \times \frac{-2000 + 105U_{2Y}}{105 \times 1000} = 0$$
$$-6550.046U_{2Y} + 122920.97U_{2Y} - 105000U_{2Y} = -2000$$
$$U_{2Y} = -0.1758871\ mm$$

Therefore, the y component of the deflection at joint 2 is $\boxed{-0.1758871\ mm}$.

Substitute $(-0.1758871\ mm)$ for U_{2Y} in the equation (2).

$$U_{4Y} = \frac{-2000 + 105U_{2Y}}{105 \times 1000} = \frac{-2000 + 10500 \times (-0.1758871)}{105 \times 1000} = -0.19493\ mm$$

Therefore, the y component of the deflection at joint 4 is $\boxed{-0.19493\ mm}$.

Comment

Step 18 of 27

Substitute $(-0.1758871\ mm)$ for U_{2Y} in the equation (3).

$$U_{2X} = \frac{26.88U_{2Y}}{110.322} = \frac{26.88 \times (-0.1758871)}{110.322} = -0.0428\ mm$$

Therefore, the x component of the deflection at joint 2 is $\boxed{-0.0428\ mm}$.

Comment

Step 19 of 27

Calculate the Reaction force at the joint by using the following relation.

$$[\mathbf{R}] = [\mathbf{K}]\{\mathbf{u}\} - \{\mathbf{f}\}$$

Substitute $\begin{bmatrix} R_{1X} \\ R_{1Y} \\ R_{2X} \\ R_{2Y} \\ R_{3X} \\ R_{3Y} \\ R_{4X} \\ R_{4Y} \end{bmatrix}$ for $[\mathbf{R}]$, $10^3 \begin{bmatrix} -70U_{2X} \\ 0 \\ 110.32U_{2X} - 26.88U_{2Y} \\ -26.88U_{2X} + 122.92U_{2Y} - 105U_{4Y} \\ -40.322U_{2X} + 26.88U_{2Y} \\ 26.88U_{2X} - 17.92U_{2Y} \\ 70U_{4X} \\ -105U_{2Y} + 105U_{4Y} \end{bmatrix}$ for $[\mathbf{K}]\{\mathbf{u}\}$ and $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2000 \end{bmatrix}$ for $\{\mathbf{f}\}$.

$$\begin{bmatrix} R_{1X} \\ R_{1Y} \\ R_{2X} \\ R_{2Y} \\ R_{3X} \\ R_{3Y} \\ R_{4X} \\ R_{4Y} \end{bmatrix} = 10^3 \begin{bmatrix} -70U_{2X} \\ 0 \\ 110.32U_{2X} - 26.88U_{2Y} \\ -26.88U_{2X} + 122.92U_{2Y} - 105U_{4Y} \\ -40.322U_{2X} + 26.88U_{2Y} \\ 26.88U_{2X} - 17.92U_{2Y} \\ 70U_{4X} \\ -105U_{2Y} + 105U_{4Y} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2000 \end{bmatrix}$$

$$= 10^3 \begin{bmatrix} -70 \times (-0.04285) \\ 0 \\ 110.322 \times (-0.04285) - 26.88 \times (-0.1758) \\ -26.88 \times (-0.04285) + 122.92 \times (-0.1758) - 105 \times (-0.1949) \\ -40.322 \times (-0.04285) + 26.88 \times (-0.1758) \\ 26.88 \times (-0.04285) + 17.92 \times (-0.1758) \\ 70(0) \\ -105 \times (-0.1758) + 105 \times (-0.1949) + 2000 \end{bmatrix}$$

$$= \begin{bmatrix} 3000 \\ 0 \\ 0 \\ 0 \\ -3000 \\ 2000 \\ 0 \\ 0 \end{bmatrix}$$

Therefore, the reaction forces are $\boxed{\begin{bmatrix} 3000 \\ 0 \\ 0 \\ 0 \\ -3000 \\ 2000 \\ 0 \\ 0 \end{bmatrix} N}$.

Comment

Step 20 of 27

Calculate local displacement matrix for element (1).

$$\begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix} \times \begin{vmatrix} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \end{vmatrix}$$

$$= \begin{bmatrix} \cos(0°) & \sin(0°) & 0 & 0 \\ -\sin(0°) & \cos(0°) & 0 & 0 \\ 0 & 0 & \cos(0°) & \sin(0°) \\ 0 & 0 & -\sin(0°) & \cos(0°) \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -0.0428 \\ -0.1757 \end{bmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{vmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -0.0428 \\ -0.1757 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ -0.0428 \\ 0.0428 \end{bmatrix}$$

Comment

Step 21 of 27

Calculate the force by using the following relation for element (1).

$$F_1 = \frac{AE \times (u_{2x} - u_{1x})}{L}$$

Substitute $1500\ mm^2$ for A, $70000\ N/mm^2$ for E, $1500\ mm$ for L, 0 for u_{1x}, and $-0.0428\ mm$ for u_{2x}.

$$F_1 = \frac{AE \times (u_{2x} - u_{1x})}{L} = \frac{1500 \times 70 \times 1000 \times (-0.0428\text{-}0)}{1500} = -3000\ N$$

Therefore, the force for element (1) is $\boxed{-3000\ N}$.

Calculate the stress in element (1) by using he following relation.

$$\sigma_1 = \frac{E \times (u_{2x} - u_{1x})}{L}$$

Substitute $70000\ N/mm^2$ for E, $1500\ mm$ for L, 0 for u_{1x}, and $-0.0428\ mm$ for u_{2x}.

$$\sigma_1 = \frac{E \times (u_{2x} - u_{1x})}{L} = \frac{70 \times 1000 \times (-0.0428 - 0)}{1500} = -2\ N/mm^2 = -2\ MPa$$

Therefore, the stress in element (1) is $\boxed{-2\ MPa}$.

Comment

Step 22 of 27

Calculate local displacement matrix for element (2).

$$\begin{bmatrix} u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix} \times \begin{vmatrix} U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \end{vmatrix}$$

$$= \begin{bmatrix} -0.832 & 0.555 & 0 & 0 \\ -0.555 & -0.832 & 0 & 0 \\ 0 & 0 & 0.832 & 0.555 \\ 0 & 0 & -0.555 & 0.832 \end{bmatrix} \times \begin{bmatrix} -0.0428 \\ -0.1757 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -0.832 \times (-0.0428) \\ 0.555 \times (-0.1757) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.061877 \\ -0.17 \\ 0 \\ 0 \end{bmatrix}$$

Calculate the force by using the following relation for element (2).

$$F_2 = \frac{AE \times (u_{3x} - u_{2x})}{L}$$

Substitute $1500\ mm^2$ for A, $70000\ N/mm^2$ for E, $1803\ mm$ for L, $-0.061877\ mm$ for u_{2x}, and 0 for u_{3x}.

$$F_2 = \frac{AE \times (u_{3x} - u_{2x})}{L} = \frac{1500 \times 70 \times 1000 \times (0 - (-0.061877))}{1803} = 3603.48\ N$$

Therefore, the force for element (2) is $\boxed{3603.48\ N}$.

Calculate the stress in element (2) by using the following relation.

$$\sigma_2 = \frac{E \times (u_{3x} - u_{2x})}{L}$$

Substitute $70000\ N/mm^2$ for E, $1803\ mm$ for L, $-0.061877\ mm$ for u_{2x}, and 0 for u_{3x}.

$$\sigma_2 = \frac{E \times (u_{3x} - u_{2x})}{L} = \frac{70 \times 1000 \times (0 - (-0.061877))}{1803} = 2.4\ N/mm^2 = 2.4\ MPa$$

Therefore, the stress in element (2) is $\boxed{2.4\ MPa}$.

Comment

Step 23 of 27

$$\begin{bmatrix} u_{3y} \\ u_{4x} \\ u_{4y} \end{bmatrix} = \begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix} \times \begin{vmatrix} U_{3Y} \\ U_{4X} \\ U_{4Y} \end{vmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.19478 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.19478 \end{bmatrix}$$

Comment

Step 24 of 27

Calculate the force by using the following relation for element (3).

$$F_3 = \frac{AE \times (u_{4x} - u_{3x})}{L}$$

Substitute 1500 mm^2 for A, 70000 N/mm^2 for E, 1500 mm for L, 0 for u_{3x}, and 0 mm for u_{4x}.

$$\begin{aligned} F_3 &= \frac{AE \times (u_{4x} - u_{3x})}{L} \\ &= \frac{1500 \times 70 \times 1000 \times (0-0)}{1500} \\ &= 0 \end{aligned}$$

Therefore, the force for element (3) is $\boxed{0}$.

Calculate the stress in element (3) by using the following relation.

$$\sigma_3 = \frac{E \times (u_{jx} - u_{ix})}{L}$$

Substitute 70000 N/mm^2 for E, 1500 mm for L, 0 for u_{3x}, and 0 mm for u_{4x}.

$$\begin{aligned} \sigma_3 &= \frac{E \times (u_{jx} - u_{ix})}{L} \\ &= \frac{70 \times 1000 \times (0-0)}{1500} \\ &= 0 \end{aligned}$$

Therefore, the stress in element (3) is $\boxed{0}$.

Comment

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Calculate local displacement matrix for element (4).

$$\begin{bmatrix} u_{2x} \\ u_{2y} \\ u_{4x} \\ u_{4y} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix} \times \begin{vmatrix} U_{2X} \\ U_{2Y} \\ U_{4X} \\ U_{4Y} \end{vmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} 0.042859 \\ 0.17574 \\ 0 \\ 0.19478 \end{bmatrix}$$

$$= \begin{bmatrix} 0.17574 \\ -0.042859 \\ 0.19478 \\ 0 \end{bmatrix}$$

Comment

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Calculate the force by using the following relation for element (4).

$$F_4 = \frac{AE \times (u_{4x} - u_{2x})}{L}$$

Substitute 1500 mm^2 for A, 70000 N/mm^2 for E, 1500 mm for L, 0.17574 mm for u_{2x}, and 0.19478 mm for u_{4x}.

$$\begin{aligned} F_4 &= \frac{AE \times (u_{4x} - u_{2x})}{L} \\ &= \frac{1500 \times 70 \times 1000 \times (0.19478 - 0.17574)}{1000} \\ &= 2000 \text{ N} \end{aligned}$$

Therefore, the force for element (4) is $\boxed{2000 \text{ N}}$.

Calculate the stress in element (4) by using the following relation.

$$\sigma_4 = \frac{E \times (u_{4x} - u_{2x})}{L}$$

Substitute 70000 N/mm^2 for E, 1803 mm for L, 0.17574 mm for u_{2x}, and 0.19478 mm for u_{4x}.

$$\begin{aligned} \sigma_4 &= \frac{70 \times 1000 \times (0.19478 - 0.17574)}{1000} \\ &= 1.333 \text{ N/mm}^2 \\ &= 1.333 \text{ MPa} \end{aligned}$$

Therefore, the stress in element (4) is $\boxed{1.333 \text{ MPa}}$.

Comment

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Verification of result as follow.

For Equilibrium condition,

$$\sum F_x = 0$$
$$\sum F_y$$
$$\sum M_{\text{node 1}} = 0$$

Write the relation for force along X direction,

$$\sum F_x = 0$$
$$3000 - 3000 = 0$$
$$0 = 0$$

Write the relation for force along Y direction,

$$\sum F_y = 0$$
$$2000 - 2000 = 0$$
$$0 = 0$$

Write the moment of the force about node 1.

$$\sum \text{M}_{\text{node 1}} = 0$$
$$57.7 \times 0 - 57.7 \times 0 - 200 \times 1.5 + 100 \times 3 + 100 \times 0 = 0$$
$$0 = 0$$

Thus, verified the results.

Comment

Was this solution helpful? 3 0